

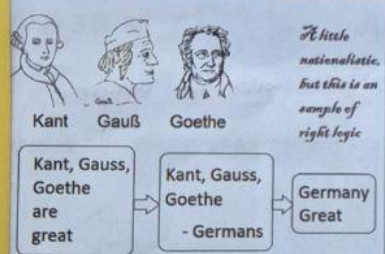
+0.1 6.6.25
 +0.1 for Handout
 25 Feb 4
 +0.1 for A

Resume of Lecture by Pr. Bob Gallager from MIT Massachusetts Institute of Technology (MIT)

George Boole (1815-1864) developed Boolean logic
 The principles of logical thinking have been understood (and occasionally used) since the Hellenic era.
 Boole's contribution was to show how to systemize these principles and express them in equations (called Boolean logic or Boolean algebra).
 Claude Shannon (1916-2001) showed how to use Boolean algebra as the basis for switching technology. This contribution systemized logical thinking for computer and communication systems, both for the design and programming of the systems and their applications.

Logic continues to be abused in politics, religion and most non-scientific areas

Logic continues to be abused in politics, religion, and most non-scientific areas.



Bad logic (abuse of logic)

model of communication system

The Mathematical Theory of Communication



2- Creating a reliable connection over an unreliable (noisy) channel that's what IT is about

emotional supply and fatty reasoning rather than truth-seeking

2 - to make the communication work well

and that's what Shannon did

+ 0.1
 3 June 25

3 - concept of information (entropy, the idea of bit (unit of info) how effectively the message can be sent)
Information Theory

2 - how to send, store and understand messages in the best way (the least space, time and fewer errors)

1+03=1.3
 for part 23



Massachusetts Institute of Technology (MIT)



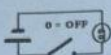
Lecture by Pr. Bob Gallagher
Boole (1815-1864) & Shannon (1916-2001)

Boolean logic
modern mathematics
system that works with

True (1) or False (0)
using AND, OR,
NOT, used in
computers to
process information



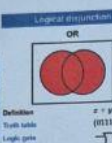
Sapere aude!



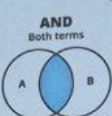
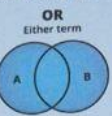
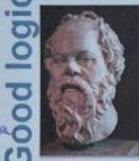
Logical addition
(disjunction)

A	B	F=A∨B
0	0	0
0	1	1
1	0	1
1	1	1

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F



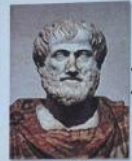
BOOLEAN LOGIC



Socrates was
a philosopher



philosophers are men



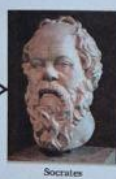
Socrates was
a man



Aristotelian logic - old system based on reasoning with words and categories
(all men are mortal), logic for thinking



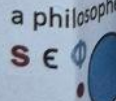
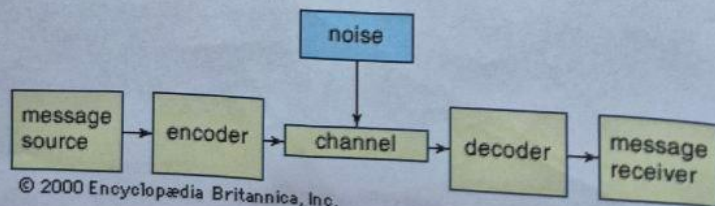
Socrates was
a man



philosophers are men



Socrates was
a philosopher



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View Model
WEB
some...

Curse
Backlog
C# 2.200
\$300
C++ 4.5
Java 3

Lab 1. Pre C
Lab 1. 1
Main (24 s
55
eye
01101
001

→ View Model View Model

→ WEB

somee.com Free Hosting ASP.NET

Cur
↓
Deezap WEB

JavaScript 2009

C# 2.2000
\$3000.

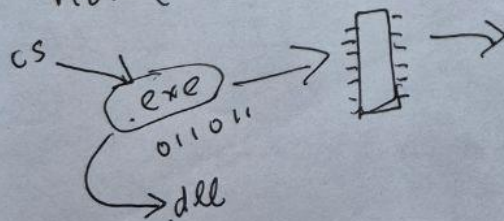
C++ 4.5000

Java 3000

Lab 1. CS

Lab 1. 1 0

Main(~~as~~ string[] args)



0.7 Sun Jan
M J++ C#

dot.net.by

A0@ dot.net.by
12p/week

C:\Windows\Microsoft.NET ... csc.exe Lab1.cs
%1

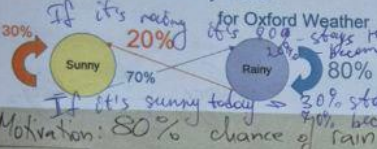
1. bat Lab1.cs

186CU
book

+0.1 to fin Grade Probability $n!$



Markoff Chain Probability Model



CHALK+TALK ink + think

take notes on the lecture yourself

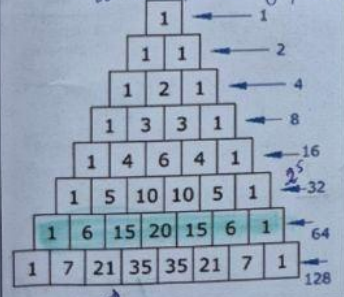
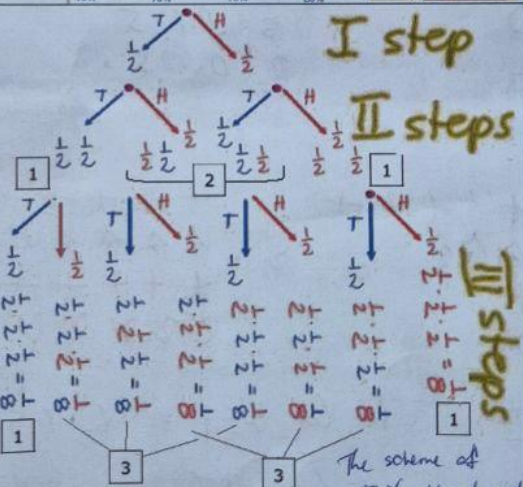
Motivation: 80% chance of rain
Let A_j be the event of rain at 9am on day j of this term, $1 \leq j \leq n$
Suppose the events A_i each have probability p , independently.

Oxford	Tue 13th	Wed 14th	Thu 15th	Fri 16th
	10°	13°	13°	11°
	70%	70%	70%	100%

School = formalism => Uni



Pascal's triangle to find patterns, solve binomial expressions, probability questions



The scheme of Tail Head probability using Pascal triangle

each number is the sum of the two numbers directly above it.

$$\begin{aligned}
 (a+b)^0 &= 1 \\
 (a+b)^1 &= a+b \\
 (a+b)^2 &= a^2 + 2ab + b^2 \\
 (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
 (a+b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\
 (a+b)^6 &= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6
 \end{aligned}$$

$$H = \sum_{x=1}^n p(x) \log_2 \left(\frac{1}{p(x)} \right)$$

$$I(x) = \log_2 \left(\frac{1}{p(x)} \right) \quad \text{Quantifying information}$$

$$H(x) = \sum_{x=1}^n p(x) I(x)$$

$$\log_2 \left(\left(\frac{1}{8} \right)^{-1} \right) = \log_2 8 = 3$$

$$\log_2 \frac{1}{0.5} = \log_2 2 = 1$$

$$\square \square \square \square - 1 \cdot 0.5$$

$$\triangle \triangle - 2 \cdot 0.25$$

$$\bigcirc - 3 \cdot 0.125$$

$$+ - 4 \cdot 0.125$$

$$\log_2 \left(\frac{1}{\frac{1}{4}} \right) = \log_2 4 = 2$$

$$\log_2 8 = 3$$

$$\begin{array}{l} + + + + + \\ \triangle \triangle \triangle \\ + \\ \heartsuit \end{array}$$

$$\begin{array}{l} + + + + + \\ \triangle \triangle \triangle \triangle \triangle \\ + + + + + \\ \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \end{array}$$

$$\log_2 \left(\frac{1}{\frac{1}{20}} \right) = \log_2 20 = 4.32$$

$$2 \cdot 4 = 8$$

$$1. \log_2 \left(\frac{1}{0.5} \right) = \log_2 2 = 1 = 1 \cdot 0.5$$

$$2. \log_2 \frac{1}{\frac{1}{10}} = \log_2 \frac{10}{1} \approx 3.32 \quad \log_2 3.3 = 1.772 \cdot 0.33$$

$$3. \log_2 \frac{1}{0.1} = \log_2 10 \approx 3.32$$

$$3.32 \cdot 0.1 = 0.3$$

$$3.32 \cdot 0.1 = 0.3$$

$$3 = 4 \cdot \frac{1}{4}$$

$$1 + 1.11 + 3.32 = 5.43$$

$$\approx 1.5$$

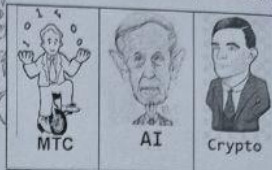
7 in binary: 0111

Pos	1	2	3	4	5	6	7
Bit	1	1	1	1	0	0	0
	p_1	p_2	d_1	p_3	d_2	d_3	d_4

p - parity bit
d - data bit

$P_1 (1, 3, 5, 7): 0+1+1+1=2$
 $P_1 = 0$ (even)
 $P_2 (2, 3, 6, 7): 0+1+1+1=2$
 $P_2 = 0$ (even)
 $P_3 (4, 5, 6, 7): 1+1+1+1=3$
 $P_3 = 1$ (odd)

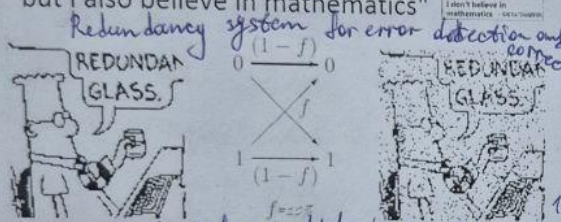
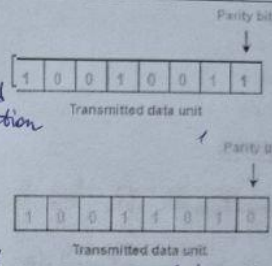
Sir Dr. D. MacKay,
 University of Cambridge
 (22 April 1967 - 14 April 2016)



7 = 000111
 (hamming code)

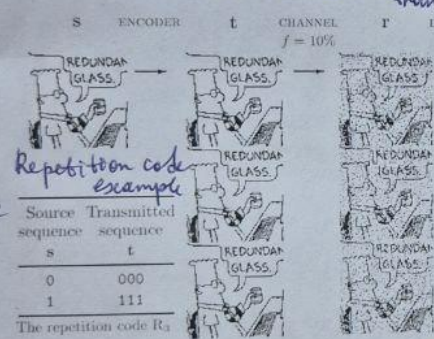


"I believe in clean energy,
 but I also believe in mathematics"



Extra piece of data (a 0 or 1) added to a group of bits (a string of a binary data) to help detect errors in data transmission.

even parity - total number of 1s is even
 odd parity - total number of 1s is odd



Source Transmitted sequence

s	t
0	000
1	111

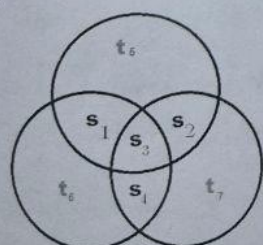
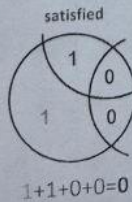
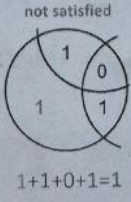
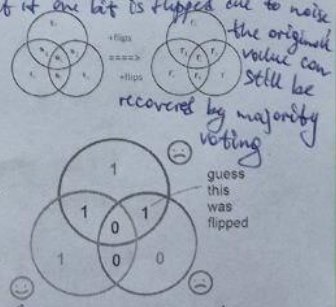
The repetition code R_3

source message s

s	0	0	1	0	1	1	0
t	000	000	111	000	111	111	000
u	000	001	000	000	101	000	000
v	000	001	111	000	010	111	000

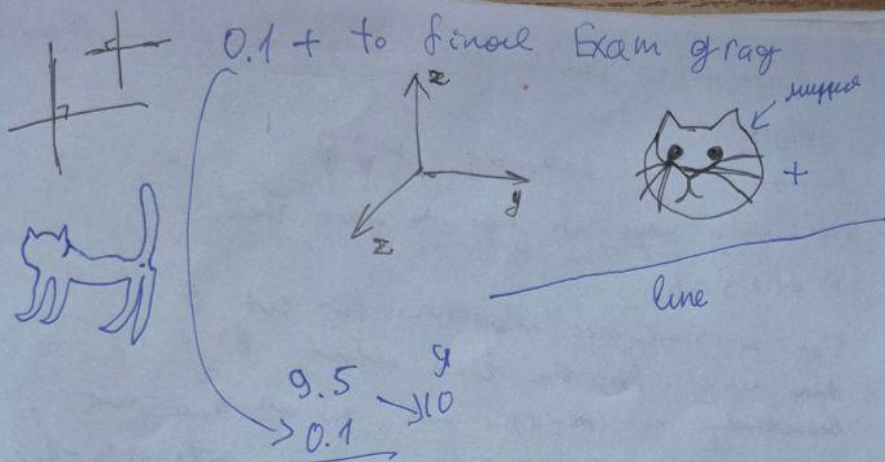
corrected errors *
 undetected errors *

7.4. Hamming code.



each codeword is 7 bits long
 of those 4 bits - actual data
 3 bits - parity (redundant) bits
 (they take positions that are powers of 2 (1, 2, 4))

Example: orig: 1
 sent as: 111
 received as: 101 (error)
 checked: 2 1s = 1



Events & probabilities

Consider an "experiment" which has a set of Ω outcomes

$w \in \Omega$, a) Tossing a coin $\Omega = \{H, T\}$ - a). coin comes up tail $A = \{T\}$

b) throwing a dice  - b). $A = \{(3, 6)\}$ - event
 $\Omega = \{(i, j), i, j \in [1, 2, 3, 4, 5, 6]\}$
 set of possible outcomes

A subset of Ω is called event.

$w \in \Omega$ is the outcome, we say that A occur if $w \in A$

Complement of A: A^c $\bar{A}$ occur when A doesn't occur

set difference: $A \setminus B = A \cap B^c$

Intersection: $A \cap B$ occur if both A & B occur

Union: $A \cup B$ occur if any A or B occur

A and B are disjoint if $A \cap B = \emptyset$ (empty set).

A and B can't occur together.

$P(A)$ of each event A. $A = \{T\}$ $A = \{(i, j)\}$

ass: Ω is finite and all outcomes are equally likely

then $P(A) = \frac{|A|}{|\Omega|}$

$|\Omega| = 2$

$|A| = 1$ $P(A) = \frac{1}{2}$

el. $|\Omega| = 6$

$|A| = 1$

$P(A) = \frac{1}{6}$

Elementary combinatorics. Arrangement of distinguishable objects.

- Suppose we have n disting. obj.  How many ways are there to order them.
 $3! = 1 \cdot 2 \cdot 3 = 6$

For n there are n options for 1st & then $n-1$ for the 2nd object & inductively $n - (m-1) -$ until in all these are

$$n(n-1)(n-2) \dots n-1 = n!$$

permutations



$$\frac{5}{8}$$

$$\frac{1}{8}$$

$$\frac{1}{8}$$

$$\frac{1}{8}$$

$$0,625 \cdot 1$$

$$0,125 \cdot 2$$

$$0,125 \cdot 3$$

$$0,125 \cdot 3$$

$$\begin{array}{r} 0,625 \\ + 0,250 \\ + 0,375 \\ \hline \Rightarrow 0,375 \\ \hline 1,625 \end{array}$$

$$1.75 \downarrow - 1,625 = 0,125 \downarrow$$

ishable

90 - \$ 70.000/y

90 - \$ 7.000/y

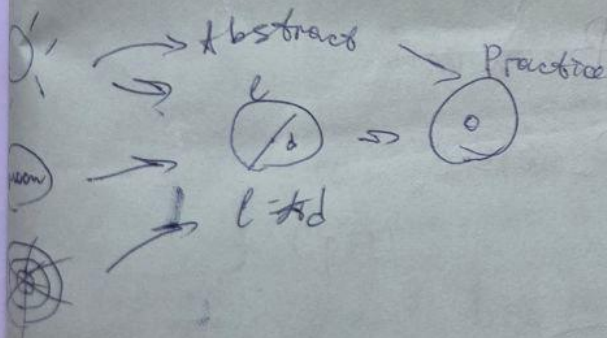
award releases course with a free certificate

h.12
strong



компонентная

+0.1 to fin

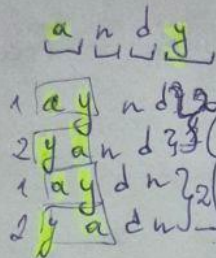


How many ways to order the letters of GALLOIS

$$n! = 6!$$

UUU UUU

$$\begin{matrix} a & n & d \\ 1 & & \end{matrix} \left. \begin{matrix} \\ \\ \end{matrix} \right\} 2$$



andy
nady
dy na

$$\left. \begin{matrix} 4! = 24 \\ 4 \\ 24 = \frac{1}{6} \end{matrix} \right\}$$

$$C_n^k = \frac{n!}{k!(n-k)!} = \frac{A_n^k}{P_k} \quad A = (n-1) \quad (n-m_1)$$

$$A_n^k = \frac{n!}{(n-m)!}$$

$$P(A) = \frac{m_1! \cdot m_2!}{n!}$$

$$n! = n(n-1)(n-2) \dots 2 \cdot 1$$

$$m_1 + m_2 = n$$

+0.1 for

$$\frac{6!}{3!}$$

$$C_n^k$$

?

Bernoulli?

Newton
Binominal

$$P_n(k) = C_n^k \cdot p^k \cdot q^{n-k} \quad (1-q)$$

ABBAT

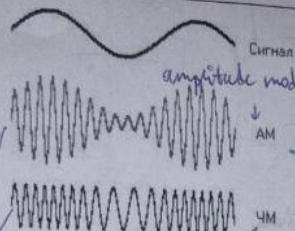
$$\rightarrow \frac{5!}{2!2!1!}$$

$$\frac{n!}{m_1! m_2! \dots m_n!}$$

+0.1 to fin Grade
 +0.2 to fin Grade 6.5.25 $T = \frac{1}{R}$



Reginald A. Fessenden
 (October 6, 1866 – July 22, 1932)

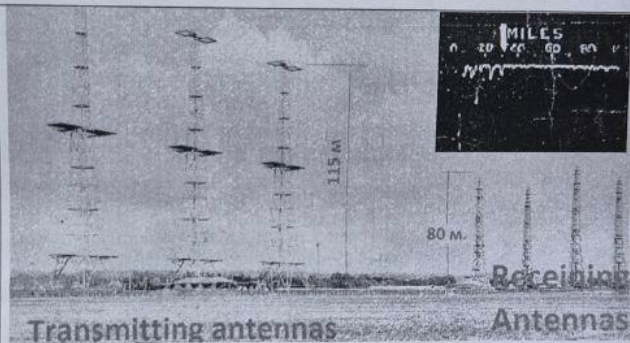


October 6, 1866 –
 July 22, 1932)
 first transmission of
 speech by radio
 (1900), and the first
 two-way
 radiotelegraphic
 communication across
 the Atlantic Ocean
 (1906)

two methods of encoding information in a signal
 frequency modulation

"Ни одна организация, занимающаяся какой-либо конкретной областью деятельности, никогда не изобретает какие-либо важные разработки в этой области или не внедряет какие-либо важные разработки в этой области до тех пор, пока она не будет вынуждена сделать это из-за внешней конкуренции." Oxford University Press. The Quarterly Journal of Economics, Feb., 1926, p. 262.

Battle of Britain
 (3 month 3 weeks)
 10.07-31.10.1940



Radar played a major role in the Battle of England

H. Nyquist



Information theory formulas

$$W = K \log m$$

Where W is the speed of transmission of intelligence,
 m is the number of current values,
 and, K is a constant.



Ralph Hartley
 (81:1888-1970)

questions about entropy

$$H = n \log s$$

$$= \log s^n$$

Bridge: deck-52 cards
 players-4 (each 13 cards)
 Hand possibilities: $S = \frac{52!}{(13! \cdot 39!)}$

$$\log_2(4) = 2$$

the number of independent card selection
 $K = n \log s$
 total entropy in bits (base-2 log)
 number of possible distinct hands

"Pacta sunt servanda"

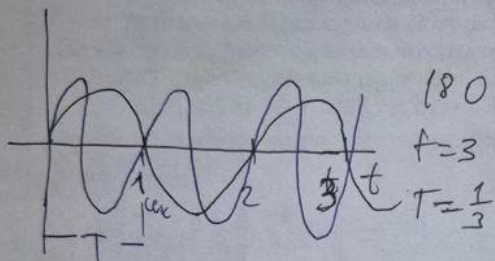
dy
 dy } 4! =
 na } 4
 24 =

$$\frac{6!}{3!}$$

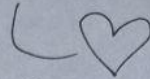
+0.3
 8
 +1
 +0.3
 10

$$\left. \begin{array}{l} f=1 \quad T=1 \\ f=2 \quad T=\frac{1}{2} \\ f=3 \quad T=\frac{1}{3} \end{array} \right\} T = \frac{1}{f}$$

Discretization
frequency



1Kz



German

60 $f=1Kz$

$T=1\text{sec}$

Run

120 $f=2Kz$

$T=0.5\text{sec}$

`<a href=" " > Home`
`<a href="Projects/1" > 1`

`</a>` | root
`<h1>`

`<a href=".../..." > Home </a>`

`<a href=".../1/" > 1 </a>`

`<a href=".../.../Projects/1" > 1 </a>`

Project

4 -- + + + +

5 ÷ 6 ♡ ♡

7 ♦

8 ♥

$\frac{4}{8} \rightarrow 50\%$

$\frac{2}{8} \frac{1}{4} \rightarrow 25\%$

$\frac{1}{8} 125\%$

$\frac{1}{8} 125\%$

} 75%

25%

$\log_2(4)$
2



AV

2 questions

50%

50%

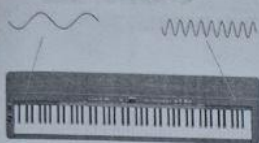
0.5*1
0.25*2
0.125*3
0.125*3

0.5
0.5
0.125*6=0.75
1.75

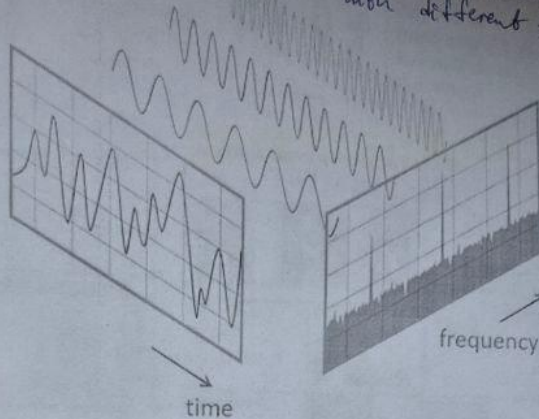
(using power
the minimum, need

* the
signal
To rec
the som
the, big

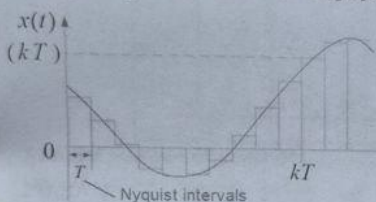
Fourier transform



mathematical method to convert a signal from the time domain to frequency domain
complex signal $\rightarrow$ simpler points (set of waves with different frequencies)



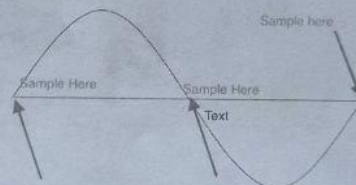
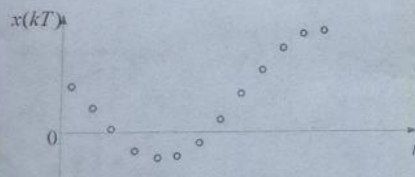
Sampling. Kotelnikov-Nyquist Theorem



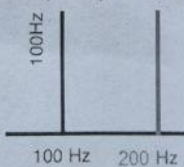
Time intervals T , through which readings $s(kT)$ are taken, are called Nyquist intervals.

Sine with period T

Sampling at $T/2$



frequency Sample



(developed fields like telecommunication, audio, image processing)

$$F_{\text{sample}} \geq 2 * F_{\text{max}}$$

$$(T_{\text{sample}} \leq T_{\text{min}} / 2)$$

Vladimir Kotelnikov

Soviet electrical engineer, radar astronomy pioneer and a member of the USSR Academy of Sciences

his work on signal sampling laid the foundation for digital signal processing, analog signals can be converted into digital form

the process of converting a continuous signal into discrete points

To reconstruct the original signal, the sampling rate must be at least twice the highest frequency present in the signal

1kHz
1000
2kHz
0.5ms

> / root

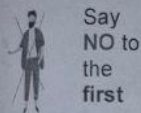
Project

< 1a >

0.5 * 1
0.25 * 2
0.125 * 3
0.125 * 3

0.5
0.5
0.125 * 6 =
1.75

important to measure, on



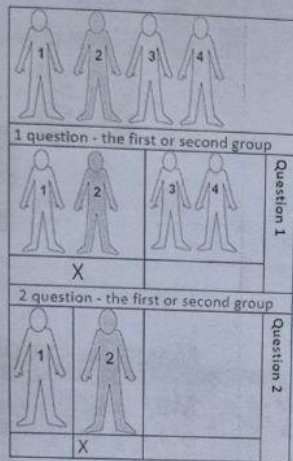
Say NO to the first



Say YES to the second if it is better than the first



Say NO to the third only if it is worse than all the others



Average number of questions = $2 \cdot 0.25 + 2 \cdot 0.25 + 2 \cdot 0.25 + 2 \cdot 0.25 = 2$

Average number of questions =

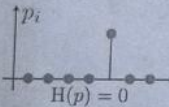
$$1 \cdot 0.5 + 2 \cdot 0.25 + 3 \cdot 0.125 + 3 \cdot 0.125$$



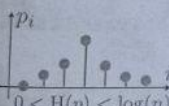
Question 1. Is this Zuckerberg?		1*0.5
Question 2. Is this Sergey Brin?		2*0.25
Question 3. Is this Stefan from BMW?		3*0.125
So Prince Saud		3*0.125

Average number of questions = 1.75

$$H(X) = -\sum_{i=1}^n p(x_i) \log_2 \frac{1}{p(x_i)} \quad \sum_{i=1}^n p(i) \log_2 \frac{1}{p(i)}$$

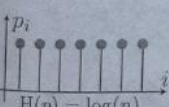


Quantifying information



$$I(x_i) = \log_2 \left(\frac{1}{p_i} \right)$$

number of bits required to encode choice



$$\sum_{i=1}^n p(x_i) I(x_i)$$

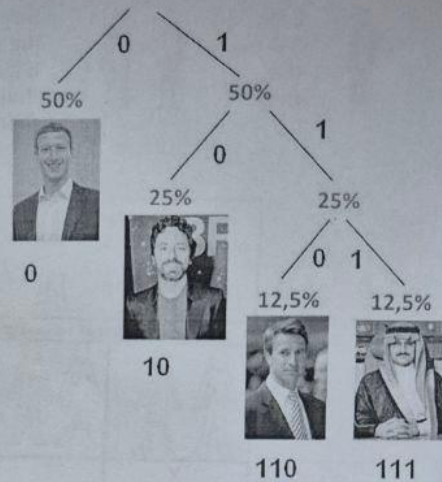


* examples with cards and questions

entropy ($H(x)$) - quantifies the average amount of information

self-information ($I(x)$) - measures how surprising the (information) measurement event may be

- If every outcome is equally likely $\rightarrow$ entropy is high (lots of unpredictability)
- If 1 outcome is much more likely than others $\rightarrow$ entropy is low (more predictability)



First-order approximation
(symbols independent but with
frequencies of Belarusian txt).

Мама мыла ра

М - 3 — 30%	1-3 М
а - 4 — 40%	4-7 а
ы - 1 — 10%	8 -ы
л - 1 — 10%	9 -л
р - 1 — 10%	10 -р

10

лла мамама р

Мама мыла ра

Ма - 2 22%	1-2 ма
ам - 2 22%	3-4 ам
мы - 1 11%	5 мы
ыл - 1 11%	6 ыл
ла, - 1 11%	7 ла
ар - 1 11%	8 ар
ра - 1 11%	9 ра

9

0. 4 6 7 3 1 9 1 6 7 3 5
ам ыл ла ам ма ра ма ыл ла ам мы
мылла рама

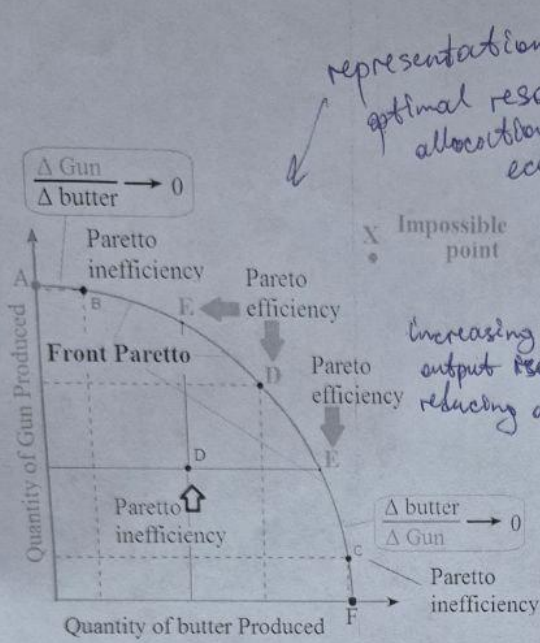


Second-order approximation (diagram (2-symbols) structure as in Belarusian)

12,5%



111



by Vilfredo Pareto

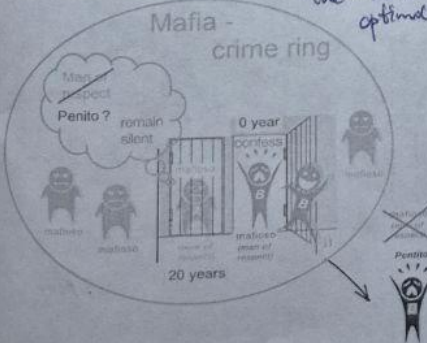
The orange sector E-D-E is the most Pareto efficient - since an increase in one indicator leads to a decrease in another.

Prisoners' dilemma

		prisoner B	
		confess	remain silent
prisoner A	confess	5 years 5 years	0 year 20 years
	remain silent	20 years 0 year	1 year 1 year

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the most optimal



Game Theory

Nash Equilibrium



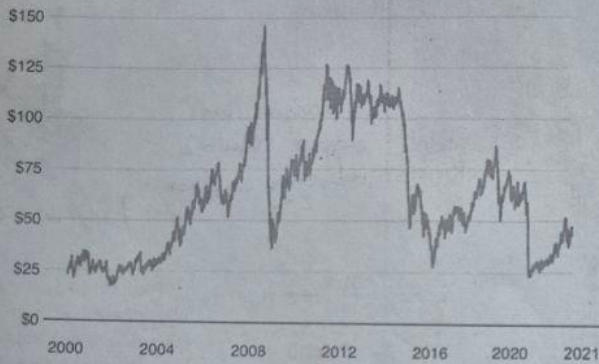
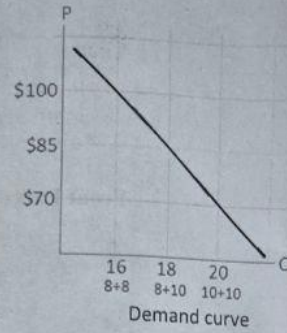
** => Nash equilibrium

		Player 2	
		Recognition;	Non-recognition;
Player 1	$H_1(x)$	1	2
	Recognition;	1 -5 *	2 0 -20
	Non-recognition;	2 -20	1 -1

-1-1
Pareto Optimality

Oil price hits 18-year low

Brent crude, US dollars per barrel



Barrel		1.		2.	
		$8 \cdot 10^6$  day		$10 \cdot 10^6$  day	
1.	$8 \cdot 10^6$ 	 \$800 millions per day  \$100  \$800 millions per day	 \$850 millions per day  \$85  \$680		
	$10 \cdot 10^6$	 \$680 millions per day  \$85  \$850 millions per day	 \$700 millions per day  \$70  \$700		
2.	$8 \cdot 10^6$ 	 \$800 millions per day  \$100  \$800 millions per day	 \$850 millions per day  \$85  \$680		
	$10 \cdot 10^6$ 	 \$680 millions per day  \$85  \$850 millions per day	 \$700 millions per day  \$70  \$700		

